THE LEVEL PAYMENT PROBLEM

1. Consider the recurrence

\[ x_n = \begin{cases} 
1.1x_{n-1} - 1000 & \text{if } n \geq 1 \\
12000 & \text{if } n = 0.
\end{cases} \]

Describe a situation this recurrence models. Use your axe to compute \( x_0 \) up to \( x_{50} \). Graph it and sketch the graph.

2. What happens to the recurrence if we let \( x_0 = 8000 \)?

3. If we allow \( x_0 \) to vary, what kinds of behaviour does the recurrence exhibit. Classify these behaviours with a simple rule. Show sample graphs. Can you choose \( x_0 \) so that the sequence remains constant? If so, explain, in common-sense terms, why this happens.

4. Let us suppose that \( x_0 \) is given, that \( a \) and \( b \) are constants, and that \( x_n = ax_{n-1} + b \). Write out the terms \( x_1 \) through \( x_5 \). Clean them up as best you can. Detect a pattern and write a formula for \( x_n \).

5. Write a recurrence for the following situation. A loan has an initial principal of \( P_0 \) and an interest rate of \( r \) per time period. An periodic payment of \( Q \) dollars is made starting with the first time period. Use the result of the last problem to write a formula for computing \( x_n \).