INTEGRAL PRACTICE

These are mixed integrals; you will be able to integrate some straight out of the box Some will require techniques such as integration by parts, substitution or partial fractions. Some of the definite integrals are improper! It is up to you to detect this.

1. \( \int_{0}^{\sqrt{\pi}} x \sin(x^2) \, dx \)

2. \( \int \frac{\sin(\sqrt{x})}{\sqrt{x}} \, dx \)

3. \( \int \frac{x \, dx}{(x-1)(x-2)(x-3)} \)

4. \( \int_{0}^{2\pi} x \cos(x) \, dx \)

5. \( \int_{0}^{4} x \sqrt{2x+1} \, dx \)

6. Revolve one hump of the sine curve about the \( y \)-axis. Find the volume of the resulting solid.

7. \( \int_{-1}^{1} \frac{dx}{x^2} \)

8. \( \int_{0}^{\pi/4} \tan^2(x) \sec^2(x) \, dx \)

9. \( \int \tan^{-1}(x) \, dx \)

10. \( \int \frac{x \, dx}{\sqrt{1+x^2}} \)

11. \( \int_{0}^{\pi/4} \sin^3(x) \cos^4(x) \, dx \)

12. \( \int x \cos^2(x) \, dx \)

13. If \( s > 0 \) is a constant, find \( \int_{0}^{\infty} e^{-sx} \, dx \)

14. \( \int \frac{x^2 \, dx}{x^2 + x - 6} \)
15. If $s > 0$ and $b$ are constants, 
\[ \int_0^\infty e^{-sx} \cos(bx) \, dx \]

16. If $n$ is a positive integer, compute 
\[ c_n = \frac{1}{2\pi} \int_0^{2\pi} x^2 \cos(nx) \, dx \]

17. \[ \int_0^1 x^2 \ln(x) \, dx \]

18. \[ \int_0^\infty xe^{-x} \, dx \]

19. For a nonnegative integer $n$, define 
\[ g_n = \int_0^\infty x^n e^{-x} \, dx. \]

Show that $g_0 = 1$. Show that for $n \geq 1$, $g_n = ng_{n-1}$; you might find integration by parts helpful. Solve the recurrence for $g$ to get a general formula for $g_n$. 